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Relativistic Description of Exclusive Deuteron Break-up Reactions
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Abstract

The exclusive deuteron break-up reaction is analyzed within a covariant approach based on the Bethe-Salpeter equation with realistic meson-exchange interaction. Relativistic effects in the cross section, tensor analyzing power and polarization transfer are investigated in explicit form. Results of numerical calculations are presented for kinematical conditions in forthcoming p + D reactions at COSY.

key words: exclusive deuteron break-up, Bethe-Salpeter equation, polarization observables

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1. Introduction: Break-up reactions of deuterons by protons receive presently a renewed interest [1, 2, 3, 4]. The investigations of break-up processes are motivated by the hope to extract directly the deuteron wave function from experimental data, supposed the mechanism described by the impulse approximation dominates at moderate values of intrinsic momenta. Pioneering experimental studies of elastic backward and inclusive D + p reactions have been performed in Dubna and Saclay [2, 3, 4]. It is found, indeed, that the impulse approximation holds in a large interval of momenta of detected protons, except for a broad shoulder at momentum of the outgoing proton (measured in the deuteron's rest frame) of about 0.3 GeV/c. Probably small corrections to the impulse approximation are sufficient to account for this shoulder. In the same experiments the polarization observables of the deuteron, such as the tensor analyzing power $T_{20}$ and polarization transfer $\kappa$ are measured. These quantities are more sensitive to the reaction mechanism, and a combined analysis of them may provide information about $S$ and $D$ components of the deuteron wave function separately. However, as in the unpolarized case, the data on $T_{20}$ and $\kappa$ exhibit systematic deviations from theoretical predictions [5, 6] in the same momentum region of the outgoing proton. A kinematical analysis of the invariant missing mass shows that this region corresponds to $\Delta$ excitations, so that additional corrections should supplement the calculations based on the impulse approximation. From this it becomes clear that an interpretation of the inclusive data in terms of a deuteron one-body momentum density becomes ambiguous. Furthermore, the typical energies in these reactions seem already too high for a non-relativistic approach. It is necessary, therefore, to describe these processes in a covariant formalism.

The measurement of proton-deuteron break-up reactions with polarized particles in an exclusive experimental setup is planned at the cooler synchrotron COSY in Jülich [1]. The experiment will detect the scattered fast proton in the forward direction in the spectrometer ANKE in coincidence with the slow backward-emitted proton. The kinematical conditions are chosen in such a way that the missing mass of the undetected particles is exactly equal to the neutron mass. Hence, this experiment offers a unique possibility of using polarized deuteron targets in combination with a polarized proton beam for extended studies of exclusive deuteron break-up processes. The coincidence measurement allows one to exclude kinematically particle production processes and, consequently, to constrain the reaction mechanism. Two important aspects in these experiments ought to be stressed here: (i) The possibility of excluding $\Delta$ excitation processes will allow for understanding better the role of the meson degrees of freedom and to clarify the origin of the above mentioned shoulder in the inclusive processes. (ii) A comparison with theoretical
predictions will provide a good test of the validity of the spectator mechanism.

Previous relativistically invariant investigations \[5, 7\] of one-nucleon exchange diagrams for inclusive and elastic D + p scattering are based on numeric solutions of the Bethe-Salpeter (BS) equation with a realistic interaction. Several authors \[8\] studied relativistic effects in the deuteron by considering the D → NN vertex within approximations to the exact BS equation and analyzed so electromagnetic and elastic hadron scattering off the deuteron. Up to now, however, a consistent investigation of polarization phenomena in exclusive processes with explicit identification of relativistic effects is still lacking.

In the present work we perform such a covariant analysis of the exclusive proton-deuteron break-up reaction. Fully covariant expressions for the cross section, tensor analyzing power \(T_{20}\) and transferred polarization \(\kappa\) are obtained within the BS formalism. The contribution of pure relativistic corrections is separated. Results of numerical calculations, utilizing the recently obtained numerical solution of the BS equation with a realistic interaction kernel, are presented for kinematical conditions available at COSY.

2. The spectator mechanism: We consider the above described exclusive break-up reaction of the type \(p + D \rightarrow p_1(0^\circ) + p_2(180^\circ) + n_3(0^\circ)\). Within the spectator mechanism approach this reaction is presented by the Feynman diagram shown in fig. 1a, where the upper and lower vertices factorize and, consequently, they can be computed separately. According to the above kinematical conditions one may consider the fast forward proton to be produced by elastic scattering of the beam proton off one nucleon in the deuteron. In fig. 1b this part of the diagram is depicted and the corresponding cross section reads

\[
\frac{d\sigma}{d\Omega^{NN}} = \frac{1}{2\sqrt{\lambda(p_p, p_n)}} \frac{\delta^4(p_p + p_n - p_1 - p_3)}{(2\pi)^2} |T_{NN}(p_p, p_n)|^2 \frac{d^2p_1}{2E_1} \frac{d^2p_3}{2E_3},
\]

where \(\lambda(p_p, p_n) = (p_p p_n)^2 - m_p^2 m_n^2\) and the invariant nucleon-nucleon (NN) amplitude \(T_{NN}\) is expressed via the truncated vertex functions \(\Gamma_{NN}\) and projectors \(\Lambda_{\alpha\alpha'}\) as follows

\[
|T_{NN}|^2 = \frac{1}{4} \Lambda_{\alpha\alpha'}(p_p) \Lambda_{\beta\beta'}(p_n) O_{\alpha\beta;\alpha'\beta'},
\]

\[
O_{\alpha\beta;\alpha'\beta'} = \sum_{\text{spins}} (u(p_3, s_3) \bar{\Gamma}_{NN} u(p_1, s_1))_{\alpha'\beta'} (\bar{u}(p_1, s_1) \Gamma_{NN} u(p_3, s_3))_{\alpha\beta},
\]

where \(u\) denote nucleon Dirac spinors and \(\alpha, \beta\) are Dirac indices (with summation over twofold indices) and the operator (2) is related to the \(T\) matrix of nucleon-nucleon scattering. To calculate the latter one in a fully covariant manner one needs an analysis of the NN vertices within an appropriate relativistic formalism, e.g., the BS approach. For the elastic scattering the NN amplitude is thoroughly investigated in ref. [9]. In our case the operator \(O\) determines the differential elastic cross section \(d\sigma/dt\) at low transferred momentum \(t = (p_p - p_1)^2\), and the contribution of the spin part in eq. (2) may be neglected.
Then the operator $O_{\alpha\beta\alpha'\beta'}$ takes the simplest possible form $O_{\alpha\beta\alpha'\beta'} \approx \delta_{\alpha\alpha'}\delta_{\beta\beta'}|A_{NN}|^2$, where $A_{NN}$ is the scalar part of the NN amplitude. Then, with $m$ as nucleon mass,

$$d\sigma^{NN} = \frac{m^2}{4\pi\lambda(p_p, p_n)}|A_{NN}|^2 dt.$$  

(3)

Now we are in the position to calculate the main contribution to the process described by the diagram in fig. 1a. Applying the Mandelstam method the invariant cross section for the break-up of a polarized deuteron with spin projection $M$ can be written as

$$2E_2 \frac{d^4\sigma}{dt \, d^3p_2} = \frac{1}{(2\pi)^3} \frac{\sqrt{\lambda(p_p, p_n)}}{\sqrt{\lambda(p_p, p_D)}} \frac{d\sigma^{NN}}{dt} \frac{(p_2^2 - m^2)}{2m} \text{Tr} \left( \bar{\Psi}_M^D(p)\hat{I} \Psi_M^D(p)(p_2 - m) \right),$$  

(4)

where $p = (p_n - p_2)/2$, and $\hat{I}$ stands either for a unity matrix in case when the outgoing proton $p_2$ is not polarized or for $(1 + \gamma_5\delta_2)/2$ otherwise ($\delta_2$ is the contracted polarization four vector). $\Psi_M^D(p)$ denotes the charge conjugated BS amplitudes introduced in ref. [10]; numerical solutions are reported in refs. [11, 12].

Eq. (4) is a rather formal result and an interpretation of different contributions is straightened in the present form. A few subtle aspects are to be mentioned:

(i) The detected proton is on mass-shell so that $p_2^2 - m^2 = 0$, and the cross section seems to vanish. However, the BS amplitude itself is singular when one nucleon is on mass-shell. To get a finite result the corresponding expressions should be evaluated analytically.

(ii) The numerical solution of the BS equation is obtained in the Euclidean space-time, where the time component $p_0$ of the relative momentum $p$ is purely imaginary. In processes under consideration $p_0$ is fixed and real. Hence, one needs either a numerical procedure for an analytical continuation of the amplitudes to the real relative energy axis (cf. [7]) or another recipe [13] for using our numerical solutions for this case.

(iii) In solving the BS equation we expand the amplitude $\Psi_M^D$ on the complete set of the Dirac matrices and obtain eight partial amplitudes for $\Psi_M^D$ [10, 14]. However, it is known [7, 8] that in such cases, where one nucleon is on mass-shell, only four partial amplitudes contribute to the deuteron observables. From eq. (4) it is not clear which amplitudes play the most important role in the process.

To tackle the above problems it is convenient to transform our representation of the partial amplitudes to the so-called $\rho$ spin classification [14, 15]. In this representation one usually adopts the spectroscopic notation for the partial amplitudes $(2S+1)L^{p1p2}$. (For the explicit form of the unitary transformation matrix between these two representations cf. [14].) In this notation it becomes immediately clear that in processes with one nucleon (say the second one) on mass-shell the relevant contribution to the cross section comes
only from four amplitudes with positive second $\rho$ spin index, i.e., $^3S_1^+, \, ^3D_1^+, \, ^1P_1^-$ and $^3P_1^-$ for which we use the short hand notation $\Psi_S, \, \Psi_D, \, \Psi_P, \, \Psi_F$, respectively.

3. Relativistic corrections: The trace in eq. (4) is evaluated by an algebraic formula manipulation code which delivers the contribution to the unpolarized cross section

$$\frac{(p_2^2 - m^2)}{2m} \sum_M T_T \left[ \tilde{\Psi}_M^D(p) \tilde{\Psi}_M^D(p)(\hat{p}_2 - m) \right] = \frac{(p_2^2 - m^2)}{8\pi E_2} (2E_2 + 2p_o - MD)$$

$$\times \left\{ \frac{1}{2} \left[ -\Psi_S^2(p_0, |p|) - \Psi_D^2(p_0, |p|) + \Psi_P^2(p_0, |p|) + \Psi_F^2(p_0, |p|) \right] \right\}$$

$$+ \frac{\sqrt{3}|p|}{m} \left[ \Psi_S(p_0, |p|) \left( -\Psi_P(p_0, |p|) + \sqrt{2}\Psi_F(p_0, |p|) \right) \right]$$

$$- \Psi_D(p_0, |p|) \left( \sqrt{2}\Psi_P(p_0, |p|) + \Psi_F(p_0, |p|) \right) \right\},$$

where another zero due to $2E_2 + 2p_o - MD = 0$ appears explicitly in the cross section ($MD$ is the deuteron mass). To handle these two zeros and the singularities in the amplitudes $\Psi_M^D$ it is convenient to introduce instead of $\Psi_M^D$ the corresponding partial vertices $G(p_0, |p|)$ that have no poles when one particle is on mass shell. For an explicit relation between partial amplitudes and partial vertices we refer the interested reader to ref. [14], where the dependence of $S$ and $D$ wave vertices upon the relative energy is shown to be smooth, contrary to the amplitudes which display a strong dependence on $p_0$. Therefore, in our calculations, we can replace, at moderate values of $p_0$, the $S$ and $D$ vertices by their values at $p_0 = 0$ with good accuracy. The $P$ vertices can be expand into Taylor series about $p_0 = 0$ up to a desired order in $p_0/m$. Then the corresponding derivatives can be computed numerically along the imaginary axis since they are analytical functions of $p_0$ [13]. After replacing the amplitudes by vertices, the zeros and singularities cancel at $p_2o_\lambda = E_2$, and the result is finite. Finally, to cast our formulae more familiar form, known from non-relativistic calculations, we introduce the notion of BS wave functions [7, 8, 14]

$$U(|p|) = F \frac{G_S(0, |p|)}{2E_2 - MD}, \quad W(|p|) = F \frac{G_D(0, |p|)}{2E_2 - MD}, \quad V_P(|p|) = F \frac{G_F(0, |p|)}{MD},$$

where $F = 1/4\pi\sqrt{2MD}$. With these definitions one gets

(i) the unpolarized differential cross section:

$$\frac{d^4\sigma}{dt d^3p_2} = \frac{MD \sqrt{\lambda(p_p, p_n)}}{2\pi^2 \sqrt{\lambda(p_p, p_D)}} \frac{d\sigma^{NN}}{dt} \cdot \frac{1}{3} \sum_M D_M(|p|)$$

$$= \frac{MD \sqrt{\lambda(p_p, p_n)}}{2\pi^2 \sqrt{\lambda(p_p, p_D)}} \frac{d\sigma^{NN}}{dt} \left\{ \left[ U^2(|p|) + W^2(|p|) - V_P^2(|p|) - V_F^2(|p|) \right] \right\}$$

$$+ \frac{2\sqrt{3} |p|}{m} \left[ U(|p|) \left( -V_P(|p|) + \sqrt{2}V_F(|p|) \right) - W(|p|) \left( \sqrt{2}V_P(|p|) + V_F(|p|) \right) \right] \right\},$$
(ii) the tensor analyzing power $T_{20}$:

$$\left(\frac{1}{3} \sum_{M} D_{M}(|p|)\right)^{\frac{1}{2}} T_{20} = \left[-W^{2}(|p|) - 2\sqrt{2} U(|p|) W(|p|) + 2 V_{A}^{2}(|p|) - V_{B}^{2}(|p|)\right],$$  

$$+ \frac{2\sqrt{3} |p|}{3 m} \left[2 U(|p|) \left(V_{A}(|p|) + V_{B}(|p|)/\sqrt{2}\right) + W(|p|) \left(2\sqrt{2} V_{A}(|p|) - V_{B}(|p|)\right)\right].$$  

(iii) the polarization transfer $\kappa$:

$$\left(\frac{1}{3} \sum_{M} D_{M}(|p|)\right) \kappa = \left[U^{2}(|p|) - W^{2}(|p|) + \frac{\sqrt{3}}{2} U(|p|) W(|p|)\right]$$  

$$+ \frac{\sqrt{3} |p|}{2 m} \left[U(|p|) \left(-\sqrt{2} V_{A}(|p|) + V_{B}(|p|)\right) + W(|p|) \left(V_{A}(|p|) + \sqrt{2} V_{B}(|p|)\right)\right].$$  

For short hand notation we introduce the deuteron structure factor $D_{M}(|p|)$ with a definition which follows from eqs. (7, 8).

A numerical analysis of solutions of the BS equation in terms of amplitudes within the $\rho$ spin basis shows [14] that the BS wave functions $U(|p|)$ and $W(|p|)$ are the dominant ones and coincide to a large extent with the corresponding non-relativistic wave functions found as solutions of the Schrödinger equation with one-boson-exchange potential. The remaining two functions $V_{A,\rho}(|p|)$ are a few orders of magnitude smaller, and for the considered processes with momentum of the backward proton $|p_{z}| = |p| \leq 0.51$ GeV/c the diagonal terms in $V_{A,\rho}(|p|)$ are negligible. Therefore, eqs. (7, 9, 11) are identified as the main contributions to the corresponding observables and they might be compared with their non-relativistic analogues. The interference terms (8, 10, 12) possess contributions from negative states and are proportional to $|p|/m$. Due to their pure relativistic origin we refer to them as relativistic corrections in the deuteron break-up reactions. Note that, when disregarding the relativistic corrections and equating the wave functions $U(|p|)$ and $W(|p|)$ to their non-relativistic analogues, our formulae (7 - 12) exactly recover the non-relativistic expressions for $\sigma, T_{20}, \kappa$ computed within the spectator mechanism [6].

4. Results and discussions: In figs. 2 - 4 we present the results of our calculations by exploiting the numerical solutions [11, 12, 14] of the BS equation with a realistic interaction kernel with $\pi, \omega, \rho, \sigma, \eta, \delta$ exchange (parameters as in table 1 in ref. [11]). The unpolarized cross section computed by eqs. (7, 8) is displayed in fig. 2, where the flux factor has been put to unity. This flux factor reflects only the dependence of the cross section on the beam energy within the spectator mechanism approach. By disregarding it one obtains the energy independent part of the cross section. Within the spectator mechanism, both $T_{20}$ and $\kappa$ do not depend on the initial energy. For the elastic neutron - proton scattering we use a fit of data [16]. The dashed curves in figs. 2 - 4 depict the
contribution of only positive waves according to eqs. (7, 9, 11), while the short-dashed curves are the relativistic corrections according to eqs. (8, 10, 12) (in fig. 2 we display the modulus of the corrections, because of a sign change), and the full lines are their sums. For completeness we also present the results of non-relativistic calculations with the Bonn potential wave functions shown as dotted curves.

It is seen in figs. 2 - 4 that the relativistic corrections are negligible small in a wide range of momenta $|p_2|$ and become significant only at $|p_2| > 0.6$ GeV/c. From this we conclude that at kinematical conditions as envisaged in COSY experiments [1] the relativistic corrections may be safely neglected since they are much smaller than the expected experimental errors. At $E_p \sim 3$ GeV one has $|p_2|_{\text{max}} \approx 0.51$ GeV/c. (Note that within the spectator mechanism the maximum value of $|p_2|$ is restricted to $|p_2|_{\text{max}} \sim 0.8$ GeV/c.) Hence, in the proposed experiments one may investigate in great detail different aspects of the reaction mechanism, or the contribution of non-nucleonic degrees of freedom like meson-exchange currents and the role of $\Delta$ isobars, without bothering about relativistic effects. Note that our expressions for the observables in exclusive break-up processes are similar to those for the inclusive ones [5, 6]. There exist an essential advantage in the proposed exclusive experiments: in inclusive processes the detected shoulder [2] in the cross section at $|p_2| \approx 0.3$ GeV/c cannot be explained within the spectator mechanism. The origin of the discrepancy is believed [6] to root in the contribution of meson production in the NN vertex. In the exclusive processes these contribution may be separated kinematically and conclusions about this problem can be settled.

The relativistic corrections eqs. (8, 10, 12) are governed by negative $P$ states in the deuteron. This can be considered as a hint that admixtures of $P$ waves within the BS approach are related to relativistic corrections by taking into account meson-exchange currents and NN pair production diagrams [17] in the non-relativistic picture. To establish a correspondence between our results and the mentioned non-relativistic calculations we estimate the contribution of the relativistic corrections by computing the $P$ wave vertices in the so-called “one-iteration approximation”. The gist of this approximation is as follows [18]: in solving the BS equation by an iteration procedure one puts as zeroth iteration the exact solution of the Schrödinger equation for $S$ and $D$ vertices and zero for other waves; then the $P$ vertices are found by one iteration of the BS equation. Our experience in solving numerically the BS equation shows that it converges rapidly for relatively small momenta $< 1$ GeV/c. That means when utilizing the exact non-relativistic solutions, after one iteration the resulting $P$ waves are not too far from the reality.

Skipping cumbersome algebraic manipulations the result for the function $V$ in eq. (6)
with a BS kernel with pseudo-scalar one-boson exchange reads as follows

$$V_{P_1,3}(|p|) = -g_s^2 \frac{2\sqrt{3}}{M_D E_2} \int_0^\infty dr \frac{e^{-\mu r}}{r} (1 + \mu r) j_1(|p|r) [N_u U(r) + N_w W(r)],$$

where $U(r)$ and $W(r)$ are the non-relativistic deuteron wave functions in the coordinate representation, and $g_s^2 \approx 14.5$ is the pion-nucleon coupling constant; $N_u = 1 (\sqrt{2})$ and $N_w = \sqrt{2} (-1)$ for $P_1 (P_3)$ waves. Introducing this result into the expression for the cross section eq. (7) the relativistic corrections is

$$E_2 \frac{d^4\sigma^{r.c.}}{dt d^3p_2} = \frac{\sqrt{\lambda(p, p_n)}}{\sqrt{\lambda(p, p_D)}} \frac{d\sigma^{NN}}{dt} \left( \frac{8 v g_s^2}{\pi^2 c^4 m^3} \int dr \frac{e^{-\mu r}}{r} (1 + \mu r) j_1(|p|r) \right. \times \left\{ U(|p|)[-U(r) + 2\sqrt{2}W(r)] + W(|p|)[2\sqrt{2}U(r) + W(r)] \right\},$$

which is similar to expressions obtained in non-relativistic evaluations of the so-called "catastrophic" and pair production diagrams in electro-disintegration of the deuteron [19]. In eq. (14) $v/c$ is the velocity of the detected slow proton, and the quantity in the large parenthesis may be interpreted as effective number of $NN$ pairs in the deuteron contributing to break-up reactions (details will be presented elsewhere [20]). From this it becomes clear that generic relativistic calculations, even in impulse approximation, contain already some specific meson-exchange diagrams, i.e., pair production currents, and one should pay attention on the problem of double counting when computing relativistic corrections beyond the spectator mechanism.

5. Summary: In summary, we present for the first time an explicit analysis of relativistic effects in exclusive deuteron break-up reactions within the Bethe-Salpeter formalism with realistic interaction kernel. Numerical estimates of relativistic effects in the cross section, tensor analyzing power and polarization transfer at kinematical conditions of forthcoming COSY experiments are performed. Relativistic corrections under these conditions are identified and found negligible. It is shown that the planned experiments can discover mainly effects related to processes beyond the impulse approximation.

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Fig. 1: Feynman graphs for the exclusive reaction $p + D = p_1(0^\circ) + p_2(180^\circ) + n_3(0^\circ)$ (a) and for the elementary NN vertex (b).
Fig. 2: The spin averaged differential cross section $E\frac{d\sigma}{dt}d^3p_2$ (with flux factor put to 1) for the exclusive proton - deuteron break-up reaction. The meaning of the curves are explained in the text.
Fig. 3: The deuteron tensor analyzing power $T_{20}$ for the exclusive proton - deuteron breakup reaction. Notation as in fig. 2.
Fig. 4: The polarization transfer $\kappa$ for the exclusive proton - deuteron break-up reaction. Notation as in fig. 2.